

Time Series Forecasting of Daily Gold Prices in Pakistan

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Abstract: This study identifies the major determinants of gold price forecasting and checks the short-run and long-run relationship along with US-Dollar prices in Pakistan. The data set covers daily observations over the period 2007 to 2018. To examine the relationship between gold price and US-Dollar price, ARIMA, ARDL and ADF model has been used. The Gold price is the variable of interest, whereas the explanatory variable is US-Dollar. The finding of the study indicates that the value of the gold price co-integrated at zero order of interaction, which means that the gold price is stationary at the level and the value of the US-Dollar is not co-integrated there for it is non-stationary at the level, but it become stationary by taking 1st difference. The result also postulated that ADF is more precise to check the short run and long run relationship between gold price and US-Dollar price. There exists long run and short run relationship between gold price and US-Dollar and also ARIMA model is used for the forecasting purpose. ARIMA model is one of the best techniques for the selection of good model. Study shows that ARIMA (3, 0, 1, 1) is a good model.

Key Words: Unit Root Test, Co-integration, ARIMA, ADF, ARDL, Gold Price, US-Dollar

Introduction

Time series is a series of data point indexed in time order. The data which shows a period of time interval is known as time series data. The technique used for time series data analysis is called time series model (Ali et al., 2016). For the prediction of historical data in time series requires a set of explanatory variables or exogenous factors (Dahlhaus, 1997). Exogenous means that regression analysis inspects elements or factors that observe external to the actual data points (Diggle, 1990). The forecasting of time series data plays an important role in the field of research (Knight et al., 2012). The most important thing about time series data is the forecasting. Forecasting of time series data plays a vital role and has become an integral part of the statistical analysis. It uses time series regression models to predict future value based on observed values (Hamilton, 1994). Time series modeling contains numerous models e.g ARIMA, ARDL, ARCH, GARCH and TAR, etc. Out of these the most prevalent for forecasting time series is Auto Regressive Integrated Moving Average abbreviated by “ARIMA”, introduced by Box and Jenking in 1970 (Victor et al., 2011). Like regression analysis, the Box-Jenkins technique requires historical data for predicting the variable. This technique assumes the clear relationship between future values with previous values and error term.

Several researchers have used ARIMA model for comparison and forecasting of time series data. ARIMA model cannot be used for long and short run relationship. To check such type of relationship autoregressive integrated moving

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average (ARDL) model or Augmented Dickey-Fuller test (ADF) is used. The ARDL approach was developed by Pesaran et al. (2001). ARDL is the most important technique used in time series data. Some of the important advantages of the model are:

It is not necessary for ARDL model that all variables co-integrated at same order of interaction. ARDL model also applied if under study variables are co-integrated of different order zero $I(0)$ or one $I(1)$, it means that ARDL model is used for different order of co-integration. For small and finite set of data ARDL model is relatively more efficient as compared to other models (Hamuda, et al., 2023).

For long run and short run relationship applied Augmented Dickey-Fuller test this test is used in this research. This test was developed by American statisticians David Dickey and Wayne Fuller in (1976). Dickey-Fuller test is used to check whether a unit root, a feature in such a case a statistical inference, is present in an autoregressive model. ADF test is one of the easy methods to check the time series data analysis. Economic and financial times series have a more complicated and dynamic structure. Therefore, simple autoregressive model and augmented Dickey-Fuller test play an important role. Gold price plays an important part in research because its value change yearly, monthly, quarterly and daily basis. Therefore, there is need to forecast gold price. Due to US dollar and any other currencies the value of gold can be changed. US dollar is the most valuable currencies in the world there for the value of gold can be compare of it. Hence positive relationship exists between gold price and US dollar price. US-dollar and gold price depend upon to other values just like inflation rate and other value. Sometime the value of gold price increase if the value of US-Dollar increases. That the price of gold increase US dollar is gone up. Sometime the value of gold increase but the value of US dollar not changed.

Gold price forecasting is an important part of economic decision making. One of the major reasons, to forecast gold price, is that consistency in the value and contain, uprising gold value even at the time of crises as compared to the money (Dickey et al., 1984). Gold assist to reduce the financial risk during inflation because the value of gold steps up with increase in inflation. It has been proved that gold has the capability to minimize the financial risk (Topal, 2010).

The price of gold also plays an important role against US dollar exchange rate. Majority of gold transitive were priced in US dollar. Gold investment becomes more attractive as compared to other investment. The increase trend allowed gold investor to speculate gold price could maintain in long run. Moreover, gold price served as an effective hedge against US dollar exchange rate transaction. Hence there is relationship exist between gold price and US-dollar price. The value of US- dollar increase then gold price value increase in Pakistan. The value of gold price depends on any other factor just like oil production, exchange rate, etc.

Our focused literature is time series studies on gold price, its determinants and forecasting performance using econometric techniques. According to the literature there is hardly any study available for gold price in relation with US-Dollar in Pakistan. Therefore, it is a need to examine the short run and long run relationship between gold price and US-dollar price by using ADF and ARDL models. ARIMA model is used for the forecasting purpose of daily gold price in Pakistan.

Kaufmann. and Winter cConducted a comparative study of annual change in gold prices for United States of America (USA) with change in inflation. The study checked the relationship between US dollar exchange rate and annual product of gold around the world. For this study, data was obtained from 1976 to 1989 regarding gold products, inflation. Applied OLS technique to study the long run relationship between variables. The result showed that long run relation exists between gold price and US Dollar prices (Kaufmann & Winters, 1989). Capie et.al, studied gold as a hedge against the dollar by using time series modeling. In this article, the data of past thirty years was used to investigate the relationship between gold price and the dollar. The result showed that gold has served as a hedge against fluctuations in the foreign exchange of value the dollar (Capie et al., 2005). Abdullah studied forecasting of gold bullion selling price through ARIMA Model. The Study showed an increasing tendency in gold bullion coin prices for this purpose annual data taken from 2005 to 2010. The analysis showed ARIMA (2, 1, 2) as the most suitable model used for gold bullion coin price (Abdullah, 2012). Bilal et.al. studied long run relationship of gold price with Karachi Stock Exchange (KSE)

and Bombay Stock Exchange (BSE). To check the long run relationship between gold price and stock exchange rate, different statistical tools were utilized e.g unit root average, Dickey Fuller (DF) test, Phillips-Person test, Johnson Co-integration test and Granger's Causality tests. The study utilized data from 2005-2011 and found a significant long run relationship between BSE and average gold price. The results also indicated the existence of causal relationship among average gold price and KSE rates and BSE rates respectively (Bilal et al., [2013](#)). Mombeini and Chamzini studied the relationship between artificial neural network (ANN) and autoregressive integrated moving average (ARIMA) model for forecasting the gold price change. In this article, the data from 1990 to 2008 years was used to estimate the models. The result indicated that ARIMA model found more powerful tool to model the gold price and forecasting than ANN model (Mombeini & Chamzini, [2015](#)). Guha and Bandyopadhyay viewed the practical application of ARIMA models and forecasted the gold price for India. This article used the data for past ten years from 2003 to 2013 regarding the trade value of gold and ARIMA (1, 1, 1) found best among the six. The ARIMA (1, 1, 1) was found the only single model among six models, which fulfilled all criteria for the forecasting of the gold prices (Bandyopadhyay, [2016](#)).

Yaziz et.al. have studied the performance of ARIMA-TGRCH with five-advance distributions i.e. Gaussian, student's-t, skewed student's-t, generalized error and skewed generalized error distribution to check the appropriate gold price model. To achieve the aim, data of gold price for 11 years was used. The study concluded that a hybrid ARIMA (0, 1, 0) –TGARCH (1, 1) found the best model for the prediction of the gold prices (Yaziz et al., [2016](#)). Rayeskumar et.al. showed the relationship among gold price, stock return, exchange rate and oil price. For this purpose, researcher utilized 3485 observation which was taken from daily basis from January 1998 to June 2011. For the analysis autoregressive and co integration techniques, were applied result showed that exchange rate is highly affected by changed of the value of gold price and as well as oil price, while stock market has fewer role in effecting the exchange rate and also study suggested that there is weak long run relationship exist gold price with stock return and long run relation occur with oil price (Shumway & Stoffer, [2011](#)). Samanta and Zadeh ([2012](#)) studied long run relationship among variables gold price, stock price, real exchange rate for dollar and the oil price of crude oil. Researcher Applied different statistical tools for analysis purpose just like co integration, common trend, factor and the spiller index. Collected daily data from January 1989 to September 2009. Result suggested that the value of stock price and gold price increase or decrease with each other but the value of gold price and exchange rate is by other variables (Samanta & Zadeh, [2012](#)). Massarrat investigated gold price has frightening spike compared to historical trend. For this comparison purpose, data was taken from January 2003 to March 2012. For this purpose, utilize autoregressive integrated moving average model because by using ARIMA model researcher easily find the best model. Result suggested that ARIMA (0, 1, 1) is the most suitable model used for predicting the gold price and it also compared by the historical trend (Hassani et al., [2015](#)).

Mehmet studied taken different variables to examine the variable that relate to the gold prices. For this purpose, variables are divided into two categories precious metal and energy to check the short-term interaction between gold price and precious metal. Utilized unit root test, co-integration test, vector autocorrelation model (VAR) and vector error correlation model (VECM). To study the short-term interaction between gold price and precious metals and employed to reveal the relationship between gold price and energy price (Eryigit, [2017](#)). The result showed that gold price has short term correlation with silver price and platinum price. The result of vector error correlation model (VECM) analysis showed that them exist long run relationship between gold price with other variables. Dilt and Kim studied relationship between gold price, dollar and oil price. Data was taken on monthly bases from January 1970 to July 2008. Utilized time series tool Granger Causality test for analysis purpose. Result showed that is negative relationship exists between the value of dollar with gold price and Oil price. The value of dollar loses the value of both commodities' gold price and oil price increase (Kim & Dilt, [2011](#)). Sujit and Kumar studied dynamic relationship among gold price, stock return, exchange rate, and oil price. Study taken daily data from 2nd January 1998 to June 2011 included 3485 observations. Different time series techniques are used for analysis purpose including vector autoregressive and co integration technique. The result showed that exchange rate is highly affected by changes of other variables, but stock market has fewer effected the exchange rate (Sujit & Kumar, [2011](#)).

Literature related gold price forecasting and also compare gold price to other commodity in yearly monthly daily etc. by using different statistical tools. ARIMA model is consider the best model of time series forecasting. Through ARIMA model the best model can easily be selected. Mostly literature study yearly monthly and quarterly gold price forecasting. No one study daily forecasting of gold price. Current study checks such repeatedly changes in Pakistan on daily basis and also checks the long run and short run relationship between daily gold price with US-Dollar price.

Motivation of the Study

Most economists and investors consider Gold as the leader commodity in stock market and its exchange rate fluctuation is less susceptibility. Gold resist, external and internal purchasing power of local currency. During all the political, financial and economic crises, gold maintains its values. The value of gold changes the value of country in global level. The fluctuation of gold is rapidly changing these changes of gold attract the investor constantly. Rate of market also depend on price of gold but in the previous year gold only buy for the protection and economic crises or inflation but today people use gold for different ceremonies like marriage etc. Then market price of gold also increases, and fluctuation can occur hourly in gold price.

Aim and Objectives

The aims and objectives of the study are as follows.

- ▶ To study the pattern of gold prices using both graphical and empirical approach
- ▶ To study the long and short run relationship between Gold and US Dollar Price
- ▶ To forecast the daily gold price of Pakistan in relation with US Dollar

Hypothesis of the Study

Several researchers forecast the value of gold price; some consider that inflation is the driver of gold price investing of gold is to hedge against inflation. Some consider that price of gold follows the random walk test it mean the value of gold change randomly. Some time gold can be considered as a monetary metal because it historically used as a currency and store of a value. Different researcher studies the value of gold yearly monthly and quarterly but not study on daily basis. Current research study about forecasting of gold value on daily basis and also compare with US-dollar price.

Significance of the Study

Gold is considered the most important commodities because of its store of value potential and investment asset status. The only commodity who retains its value any type of crises especially financial and economic. It not only plays an important role in financial assets as an international currency reserve but also play an important role in the stabilization of international money market significantly. The original value of gold over the year has its capacity to hold in term of actual purchasing power. Gold increases the purchasing power at time, when currencies have been weak. Different market investor lost faith of sudden and frequent changes in gold price because of loss of faith by the investor, the economic activities down which has some serious implication for the economy. In Pakistan economy the gold prices have a strong relationship with US-Dollar. e.g. after the 2018 election the dollar decreased three rupees, and it is observed that gold price also decreased. Before election 2018 the value of Gold was 5054.67 grams but after election the value decreased 4778.01 grams. It means that gold price has strong relation with US-Dollar price.

Most of the literature the used ARIMA model for forecasting as stated earlier that ARIMA model is primary design for forecasting. The current study remains also use ARIMA model to forecast the daily gold price in Pakistan. Current study also uses ARDL and ADF model to check the long and the short run relationship between daily gold price in Pakistan and US-Dollar price.

Methodology

The goal of our study is to examine the determinants of time series forecasting of daily gold price in Pakistan. A lot of literature available on gold price but not found a significant literature for Pakistan in daily basis which empirically examined the factors affecting on daily gold price and US-Dollar both in short as well as in long run and also forecasting of gold

prices. For short-run and long-run analysis ARDL and ADF are used and ARIMA model is used for the forecasting performance. This chapter show economic relationship among variables, availability of data, sources of data and steps to examine the short and long-run association and forecasting performance of daily gold price in Pakistan.

Material and Methods

The methodology will be used for the forecasting of daily gold prices of Pakistan and check the long run and short run relationship between gold price and US-dollar. The materials and method discuss as follow.

Data

For the empirical analysis of the gold prices, secondary data have used. The data regarding the daily basis gold prices of Pakistan and US-dollar obtained from official publications of State Bank of Pakistan for the period of January 2007 to 2018. The gold price is used as the response variable and US-dollar consider explanatory variable. The data will be analyzed in Eviews10 software package.

Methodology

Numerous methods have been used to study the daily gold price and US-dollar price. These studies include autoregressive integrated model (ARIMA), augmented dicky fuller test (ADF), co-integration approach, optimization models in the analysis of daily gold price and US-dollar price and to check long and short run relationship between. Several problems have been occurring like non-stationary problem. Investigate short run and long run association between integrated variables, co-integration test and estimation of ECM are available.

Following statistical methodology will be used in this study.

Time Series Modeling

Time series modeling is a computational procedure to model any pattern e.g. trend, seasonality etc. that occurs in the time series observations and to estimate the unknown parameters of the model. For the forecasting purposes expertise is required to predict long term variations in a system (Hillmer & Tiao, 1982) Different statistical tool are available for forecasting and to check the long and short run relationship. In this study, Box-Jenkins (ARIMA) approach is used to check a good model. Through ARIMA model first we checked the stationary of the data if our data is non-stationary its necessary to convert stationary. ADF is the best approach to check the stationary of data and ARIMA is used for the forecasting purpose. For the short-run and the long-run relation between gold price and US-Dollar we use ARDL and ADF test.

Autoregressive Integrated Moving Average Model (ARIMA) Model

ARIMA model is the generalized case of ARMA (Auto Regressive Moving average). In Econometrics and Statistics, an auto regressive integrated moving average is used for forecasting and predicting of economic variables. In this method, the response variable will be extracted from historical data set. Unlike in regression analysis, this technique can be applied when the data shows evidence of stationarity. If the data is non-stationary, an initial differencing step may be applied to make data stationary. There are numerous causes why an Autoregressive Integrated Moving Average (ARIMA) model is better than to multivariate regression and joint time series analysis. The joint finding in multivariate regressions and time series investigation is that the error-term is associated with the values of their own lagged. This autocorrelation violates the regression theory assumption that the current value of the error does not depend on the previous error values. The key problems related with autocorrelation are:

Basic time-series analysis and regression are not so longer effective amongst the dissimilar linear estimators. Though, as the previous residuals which help for prediction of current residuals, so we take benefit from this type of information to well forecast of the response variable through autoregressive integrated moving average. Standard errors which calculated by using the time series analysis and regression are not accurate and generally understand. If there include lagged value of the response variables the estimations coefficient of repressors is not unbiased and also not consistent, but these problems are fixed in ARIMA model. The approval of the autoregressive integrated moving average

model is in line for its statistical properties and its famous Box Jenkins procedure in the model structure progression. In addition, numerous exponential smoothing models can be executed by ARIMA models. While ARIMA models are relatively flexible which represent different kinds of time-series, that is autoregressive, moving average and combine both are autoregressive moving average (ARMA) series, their primary constraint is the pre-assumed linear form of the model. i.e., a linear relationship construction is presumed amongst the values of time-series and for that reason, no non-linear forms can be captured by the ARIMA model (Zhang, 2001).

ARIMA Model

The autoregressive integrated moving average model, the forthcoming value of a variable is presumed to be a linear function of numerous previous observations and randomize errors. The autoregressive integrated moving average model is commonly written as an ARIMA (p, d, q) model where parameters p , d , and q are positive integers. Autoregressive integrated moving average model is an important part of the Box-Jenkins technique to time-series modeling. The autoregressive integrated moving average model can be written as following ARMA (p, q) mode

$$Y_t = \alpha_1 y_{t-1} + \alpha_2 y_{t-2} + \dots + \alpha_p y_{t-p} + e_t + \beta_1 e_{t-1} + \dots + \beta_q e_{t-q} \quad (3.1.1)$$

Where e_t is the error and is the real value at time period, correspondingly; ($i=1, 2, 3, \dots, p$) and ($j=0, 1, 2, \dots, q$) parameters of the model q and p are integers and frequently represents orders of model. The error, presumed to be identically and independently distributed with zero mean and constant variance equation (3.1.1) requires several essential special cases of the autoregressive integrated moving average family of models. If the value of q is zero, then equation (3.1.1) becomes an autoregressive (AR) model p order. When $p=0$, the model converts to a moving average (MA) model of q order. The Box-Jenkins method comprises three iterative stages first is the identification of model, second is diagnostic checking and third is estimation of parameter. The first step, the identification of model is that if a time-series is created from an autoregressive integrated moving average procedure; it should have some theoretical serial correlation properties. By identical patterns of the experiential serial correlation with the hypothetical ones, it is frequently probable to recognize one or numerous potential models for the given time-series. Box and Jenkins projected partial autocorrelation function and autocorrelation function for lag selection of autoregressive integrated moving average model. The last stage of model structure is to check diagnostic test for model acceptability. This is fundamentally to check the assumptions of the model. Numerous statistics of diagnostic test and residuals plots is used to examine the good fit of the data. If model is not accepted, means not fulfilled the assumption of the theory than a new uncertain model should be known, which is again followed by the steps estimation of parameter and model confirmation. This three-stage model structure procedure is usually repeated numerous times up to acceptable model is lastly nominated. Now the lastly model which selected can be used for the forecasting purposes. With the special effects of Box Jenkins, the ARIMA specification is most popular univariate methods in the predicting research and practice (Zang, 2003).

The Box-Jenkins Methodology (BJ)

Statisticians George Box and Gwilym Jenkins proposed the Box-Jenkins approach. They introduced ARMA or ARIMA to find best fit model based on previous values of time series for the purpose of forecasting.

Step 1 (Identification): Box-Jenkins methodology requires stationarity of data. After making the series stationary, then run correlogram of stationary data. And identify the suitable values of p , d and q making use of Autocorrelation function (ACF) and Partial autocorrelation function (PACF).

Step 2 (Estimation): After the determination of p , d and q , estimate the coefficient of the model using Maximum likelihood or Least-Square method of estimation.

Step 3 (Diagnostic Checking): In residual diagnostic formal examination of every model would be check. There are different techniques to check this assumption in this study actual versus fitted residual plot and Correlogram Q-statistic would be used to decide whether the fitted model support the white noise assumption or not? If the assumption was not supported, we will have to start over.

Step 4 (Forecasting): Box-Jenkins ARIMA is popular for its forecasting accuracy. This technique provides reliable forecast than those obtained from other classical Econometrics modeling. In this study, forecasting would be done by graphical

method. And the accuracy of the forecast would be judged by Root Mean Square Error and Mean Absolute Percent Error. Minimum the indicated errors the more effective would be the forecast.

Augmented Dicky-Fuller Test

Stationarity is the pre-requisite in econometric analysis of time-series data. A series will be considered white noise if and only if the mean is zero, the variance is constant and covariance is zero (Deadman & Wojciech, 1992). If any of the white noise property doesn't holds then series is non-stationary. Such non stationarity is one of the contentious issues, while we are dealing with time series analysis. ADF tests are commonly used to check the Stationarity level of the series. In case of non-stationary, series can be transformed into stationary by de-trending. A suitable way out of detrending is by using first differences somewhat levels of the variables. Sometimes, the series is stationary by more than once. The non-stationary series which are transformed into stationary by differencing 'time is supposed to be integrated of order d as seen is approximately $I(d)$, (Engle & Granger, 1987). If the series becomes stationary at level, it is known to be integrated of order zero $I(0)$. Before proceeding with Co-integration analysis, there is need to check the possibility of unit-root in the series whether the given series is stationary or non-stationary. If the series is integrated at order zero, meaning the series is stationary at level. Then there is no need to do Co-integration analysis but if series are non-stationary then we can apply different Co-integration techniques. While we are dealing daily series then Augmented Dicky Fuller (ADF) test is the first choice of econometricians to check the possibility of unit roots (Deadman & Wojciech, 1992). Several tests are also available for testing unit root like Phillips-Perron (PP), Kwiatkowski-Phillips-Schmidt-Shin (KPSS) but ADF test has been preferred in the literature over PP and KPSS tests because later are more sensitive to time series. ADF is the generalized form of Dicky Fuller (DF) test. Therefore, ADF test involves followings regression:

$$\Delta Y_t = \beta_0 + \beta_1 y_{t-1} + \sum_{i=1}^k \beta_i \Delta y_{t-i} + \varepsilon_t \quad (3.2.1)$$

Whereas, the variables, y^* is to be checked for the stationarity, the first difference operator, the error term and k are the order of lags to check the possibility of autocorrelation. The null (H_0) and alternative (H_1) hypothesis can be shown as:

H_0 : There exists unit root in the series

H_1 : The given series is stationary

For the rejection of null hypothesis, probability value must be less than 0.05 or statistically significant (Fuller, 1976).

Co-Integration

It is possible that a non-stationary series can be transformed into stationary if we form the linear combination of both stationary and non-stationary series. A certain linear combination of two and more variables becomes stationary. Overall, a specific linear combination of two or more $I(d)$ variables may become an $I(b)$ variable, $b \leq d$. Therefore, the no stationary series will be considered as cointegrated. The Cointegration relationship demonstrates the presence of long run equilibrium relationship among the variables. Hence the long-run linear association among the variables is as follows:

$$y_t = b_0 + b_1 x_{1t} + b_2 x_{2t} \dots \dots \dots b_m x_{mt} + v_t \quad (1)$$

Whereas y is the response variable and $X_1, X_2, \dots, X_m$ are explanatory variables. When in the above equation, two variables seem to appear, and then it is necessary that both Y_t and X_t should be integrated at same orders. Whereas if there are two or more independent variables, then integration order of the response variable should not be greater than the order of any of the independent variables. It is also considered that either none or at least two independent variables integrated to the same order greater than the order of the response variable. The above equation can be estimated by using OLS estimation technique. However, if the Equation (1) identifies a long run association, then use of ADF test on the estimated residuals. in Equation (1) should show that the errors have a lower order 'b' than the order 'd' of, $X_1, X_2, \dots, X_m$ and y . In this case, the $b_1, b_2, \dots, b_m$ represent the long-run elasticities of the response variable 'y' with respect to the corresponding explanatory variables. When the co-integrating relationship is established, then the final step of the co-integration analysis is the creation of an Error Correction Mechanism. Co-

integrating variables are probable to re-establish themselves to their long-run equilibrium whenever there is a drift, and Error Correction Mechanism deals with short-run behavior. The Error Correction Mechanism is expected using the following linear equation:

$$\Delta y_t = c_0 + \sum_{i_1=0}^{n_1} c_{1i_1} \Delta x_{1(t-i_1)} + \sum_{i_2=0}^{n_2} c_{1i_2} \Delta x_{1(t-i_2)} \dots \dots \dots \sum_{i_m=0}^{n_m} c_{1i_m} \Delta x_{1(t-i_m)} + \sum_{j=1}^p c_j \Delta y_{t-j} + \mu_t \quad (3.4.1)$$

Where $n_1, n_2, n_3, \dots, n_m$ and 'p' are the lagged terms of their similar variables, chosen to make the residual term, white noise. Now first we check that these variables are stationary or non-stationary, if non-stationary then the order of integration, and then the long-run relationship exists or not (MacKinnon, 1996).

Auto Regressive Distributed Lag (ARDL) Model

Autoregressive distributed lag modeling methodology established by Pesaran and Smith (1998), and Pesaran et al. (2001). This technique becomes popular due to its several advantages like it is a single equation cointegration procedure. Other cointegration procedures like Engle and Granger (1987) suitable only for two variables and Johansen and Juselius (1990) can only be applied on the series also which are integrated for the same order but more than two variables and correct for large sample dataset. ARDL method is appropriate irrespective of the independent variables that are integrated at the same order or not (Hamuda, 2013). It is also appropriate for finite sample data. If the variable order of integration, is I (2), then the ARDL model is not applicable and the calculated F statistics, as created Pesaran et al. (2001) and Narayan (2005) are not valid for longer (Narayan & Narayan, 2005). ARDL have unbiased results for small sample whereas Engle and Granger produce biased results for small samples (Moynihan et al., 2000). Another advantage of the ARDL method is that evaluation is probable even when the regressors are endogenous and is sufficient to concurrently 30 accurate for residual autocorrelation (Tang et al., 2004). To check the long run relationship the test Wald or F statistics was used developed by Pesaran (2001). The F statistics distribution used non-standard under the H_0 that there is no long run association among the dependent and independent variables and alternative as H_1 exists long run relationship, whereas the repressors are I (0) and or I (1). An ARDL model is depicted below

$$y_t = \mu + \sum_{i=1}^p \beta_i y_{t-i} + \sum_{i=1}^q \gamma_i x_{t-i} + \mu_t$$

Where y_{t-i} and x_{t-i} are variables which are both stationary and has no autocorrelation (white noise).

Lagrange Multiplier (LM) Test

Most of the time series data show autocorrelation of the disturbances across periods due to omitted factors or inclusion of variables which are correlated across series (Guilkey & Schmidt, 1989). The test available for the detection of autocorrelation based on the principal that error terms are autocorrelated. Box and pierce's Test and Lung's Refinement (1970) or Q Test, the Durbin Watson Test (1970) and Lagrange Multiplier Tests are used for the detection of autocorrelation but here we will use Lagrange Multiplier test because it is superior to other tests in the sense that Durbin Watson test can only be applied on the first order autoregressive process and Box and Pierce's test become less powerful than LM test when the null Hypothesis (H_0) is rejected but asymptotically both LM and Box and Pierces test are equal. LM test was introduced by Breusch and Gogfrey in (1978). Both authors worked independently to develop LM Test against the autoregressive or moving average process. The LM test is accurate for testing higher order autocorrelations in dynamic models, but it can only apply two tailed not on one tailed. The one-sided tests were introduced by Majumder and King (1989), Basak, Roise and Majumder (2005, 2008) but these tests only tested autocorrelations in linear regression models and cannot in the dynamic models (Engle & Granger, 1987), the null and alternative hypothesis are as follows:

H_0 : There is no autocorrelation.

H_1 : There is autocorrelation.

Diagnostic Tests

For diagnostic checking behind tests will be used Jarque Bera Test for normality, Breusch Pagan/Godfrey Lagrange Multiplier Test for Heteroscedasticity and Breusch Godfrey Lagrange Multiplier Test for Autocorrelation and CUSUM, CUSUM-Square tests for stability will also be applied.

Empirical Model

Now take our empirical variable for the ARDL model. The co-integration association for gold price equation is expected which used bounds test, lagged terms shows the long run part and difference terms showing short run dynamics and ECM (Error Correction Mechanism). shock occurs Indicate speed of adjustment

Bound Testing Procedure

For analyzing the long run relationship, the following null and alternative hypothesis will be observed:

$H_0 : \theta_0 = \theta_1 = \theta_2 = \theta_3 = \theta_4 = 0$ (No long run relationship exists)

$H_1 : \theta_0 \neq \theta_1 \neq \theta_2 \neq \theta_3 \neq \theta_4 \neq 0$ (long run relationship exists)

Narayan & Narayan (2005) formulated two critical values which show $I(0)$ for lower bound and $I(1)$ for upper bound. For level of significance, if the calculated value which calculated by F-statistics is greater than $I(1)$ upper bound then cointegration exists, when the calculated value is less than lower bound $I(0)$ then the cointegration not present in the model and when the calculated value fall within the $I(0)$ and $I(1)$ then inconclusive. To establish the ARDL model which is good fit, so conduct the stability and diagnostic tests. Diagnostic test studies normality of the model, autocorrelation, the functional form and heteroscedasticity. Where lags selection depends upon the Akaike Information Criterion and Schwarz Bayesian Criterion.

Stability Tests

Since the earliest days of macro econometric analysis, researchers have been concerned about the appropriateness of the assumption that model parameter remains constant over long periods of time. This concern is also central to the so-called the Lucas critique which has played a central role in shaping macro econometric analysis in the last thirty years. Lucas stressed the point that the verdict models of financial agents are hard to define in terms of stable parameterizations, just due to changes in strategy may change these verdict models and their respective parameterization. These points of view emphasize the prominence of using structural stability tests as diagnostic checks for macro econometric models (Boldea & Hall, 2013).

Empirical Analysis

Augmented Dicky-Fuller Test

For performing Co-integration analysis, it is needed to check the order of integration of all variables included in the model. The stationary of the series being checked by Augmented Dicky-Fuller (ADF) test empirically in common practice. The following table I represents the results of ADF.

Table I

ADF Test Results

Variables	Order of integration	Result	Level		1 st difference	
			t-statistics	p-value	t-statistics	p-value
Gold price	$I(0)$	Stationary	-50.76307	0.0001		
US-Dollar	$I(0)$	Stationary	-1.483452	0.5421	-74.74569	0.0001

Table-I the results depicted that variable gold price is integrated at level $I(0)$ and the $P = 0.0001$ which show statistically significant mean that it also indicates the stationary of gold price. i.e. US-dollar is not integrated at level and also $P = 0.5421$ which indicate that the variable is insignificant and also indicate nonstationary of variable therefore take

the difference of variable to convert the non-stationary to stationary at first order of integration. The $P=0.0001$ which become significant integration of order one $I(1)$ and that satisfy the conditions under which ARDL can be used. The condition that the dependent variable is integrated of zero order. US-dollar is cointegrated of $I(1)$. Variables show the condition that the variables are not cointegrated at same order. The third condition that none of the variables are integrated of order two which also satisfied. Fourthly, our sample data is a large sample is I so we can apply the Autoregressive Distributed Lag Model.

Figure 1 shows that gold price becomes stationary for the level and no need to take difference of gold price. It indicates that there is no change in mean price and has constant variation throughout the time period. That series is stationary at level no need to take the difference of variable. The variable is co-integrated at order zero $I(0)$.

Figure 1

Diagram Shows the Series is Stationary at Level

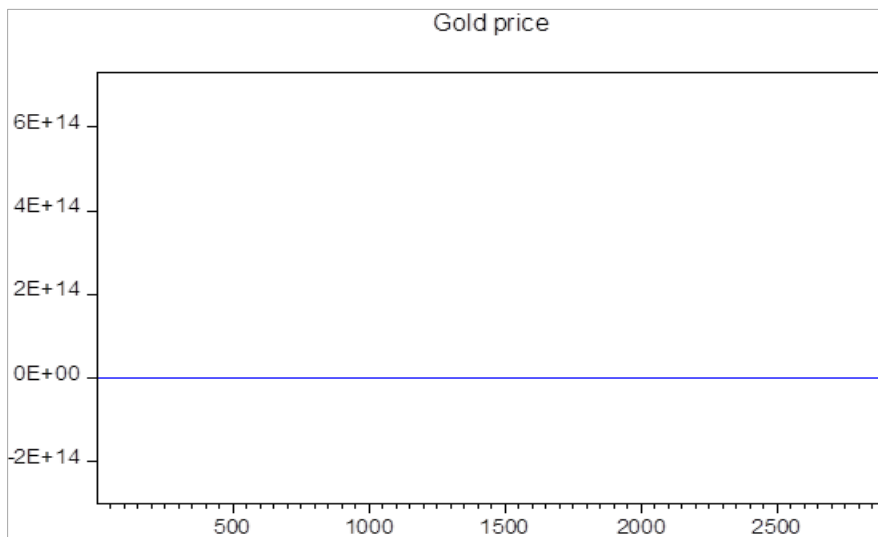


Figure 2 shows time on x-axis and daily price on y axis. The pattern reveals that there are many ups and downs in the US-Dollar. Long-term decreasing pattern of daily price of US-Dollar indicates non-stationary. It means that variable is not co-integrated therefore it is necessary to convert the non-stationary to stationary.

Figure 2

Diagram Shows that the Series is Non-Stationary at Level

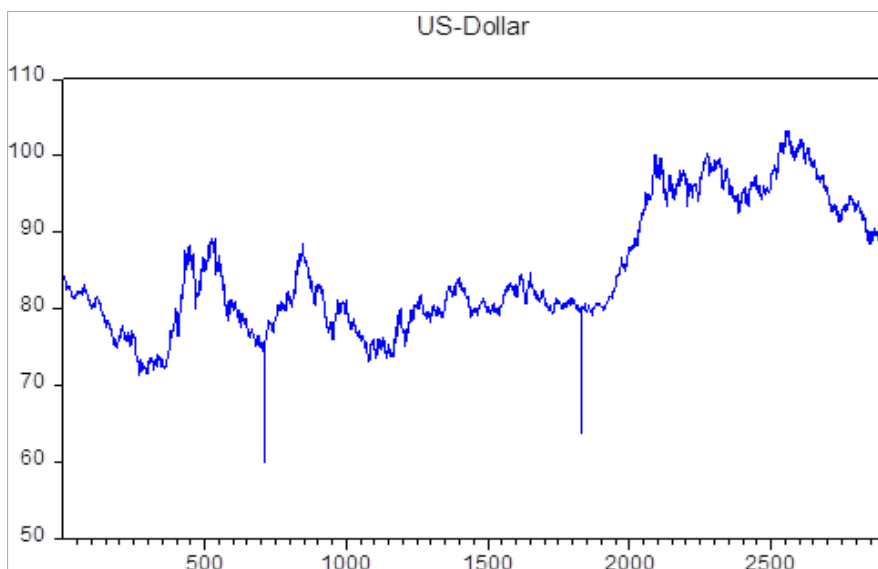


Figure 3

Diagram Show that the Series is Stationary at 1st Difference

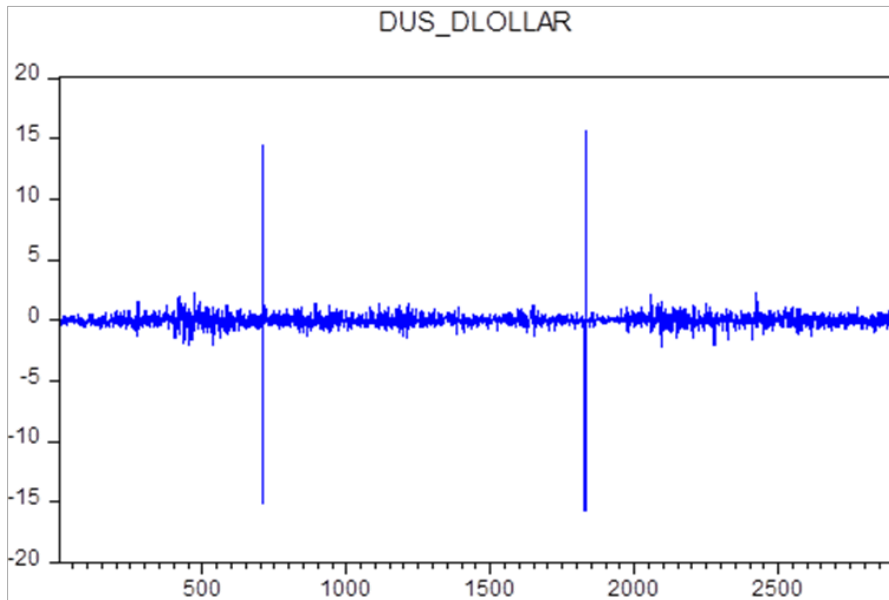


Figure 3 shows line diagram of the daily price of US-dollar, by using the 1st difference. It indicates that there is no change in mean price and has constant variation throughout the time period so, the series is stationary at 1st difference.

Table 2

Unit Root Test

Variables	Order of Integration	Test Statistic	5%P-Value	Result
Gold	I (0)	-50.76307	0.0001	Stationary
US Dollar	I (1)	-74.74569	0.0001	Stationary

In order to find the stationary of the variables of this study, unit root test is applied to the dataset. The ADF test confirms the stationary of the data on the basis of critical and calculated value. The results show that only dependent variable gold price returns are stationary at level and co-integrated because it has greater absolute test statistics value than its 5% critical value. US-Dollar is not co-integrated at level and non-stationary at zero order but stationary at 1st order of co- integration to their 5% critical values.

Co-integration

According to ADF model the variable is stationary at first difference i.e., US-Dollar is integrated of order one, I (1), and gold price integrated at level or of order zero i.e. I (0). In such a situation the long-run relationship between US-Dollar and gold price has been analyzed by ARDL Bound test. The results of final ARDL model are represented below:

Table 3

Co-Integration Test

Variable	Coefficient	Standard error	p-value
C	0.157246	0.082513	0.0568
GOLD (-1)	0.104065	0.018710	0.0000
GOLD (-2)	0.086302	0.018786	0.0000
GOLD (-3)	0.079345	0.018812	0.0000
GOLD (-4)	0.079607	0.018841	0.0000
GOLD (-5)	0.079136	0.018892	0.0000
GOLD (-6)	0.066796	0.018944	0.0004

Variable	Coefficient	Standard error	p-value
GOLD (-7)	0.068712	0.018974	0.0003
GOLD (-8)	0.060900	0.018997	0.0014
GOLD (-9)	0.052941	0.019003	0.0054
GOLD (-10)	0.054981	0.019004	0.0038
GOLD (-11)	0.046246	0.018988	0.0149
GOLD (-12)	0.033693	0.018954	0.0756
GOLD (-13)	0.026957	0.018928	0.1545
GOLD (-14)	0.026032	0.018879	0.1680
GOLD (-15)	0.034637	0.018828	0.0659
GOLD (-16)	0.026465	0.018781	0.1589
GOLD (-17)	0.029804	0.018704	0.1112
GOLD (-18)	0.032731	0.018596	0.0785
DOLLAR (-1)	-0.373881	0.191868	0.0514
DOLLAR (-2)	-0.940280	0.208217	0.0000
DOLLAR (-3)	-0.967032	0.211351	0.0000
DOLLAR (-4)	-0.579722	0.209318	0.0056
DOLLAR (-5)	-0.249007	0.192329	0.1955
DOLLAR (-1)	-0.015051	0.019315	0.4359

Figure 4

Figure shows the Results of Stability Tests Applied on ARDL Mode

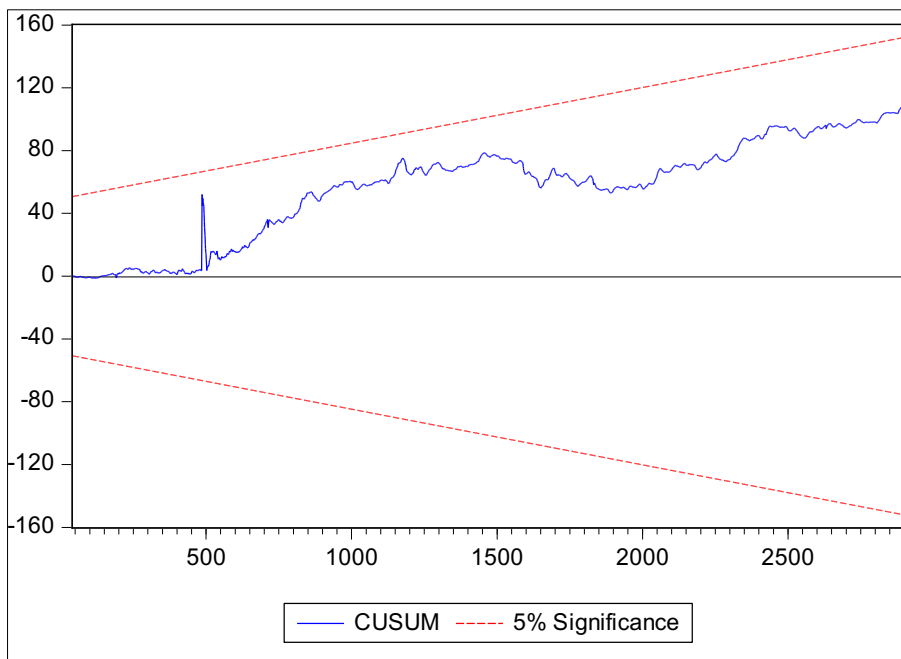


Figure 4 Straight lines represent critical bounds at 5 percent significance level. The design of the stability test of the model is available in above figure (CUSUM). The CUSUM are designed against the critical bound at the 5 percent significance level. The results displayed that the model is sable in the meantime the critical bounds at 5 percent fell in between 5 percent lines. The result showed that the model is stable since the critical bounds at 5 percent fell in between the two 5 percent lines. Also, there is no autocorrelation in the model

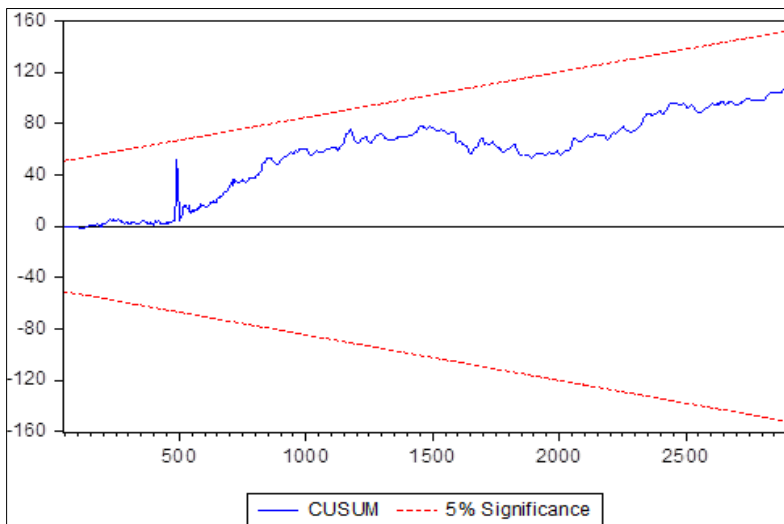
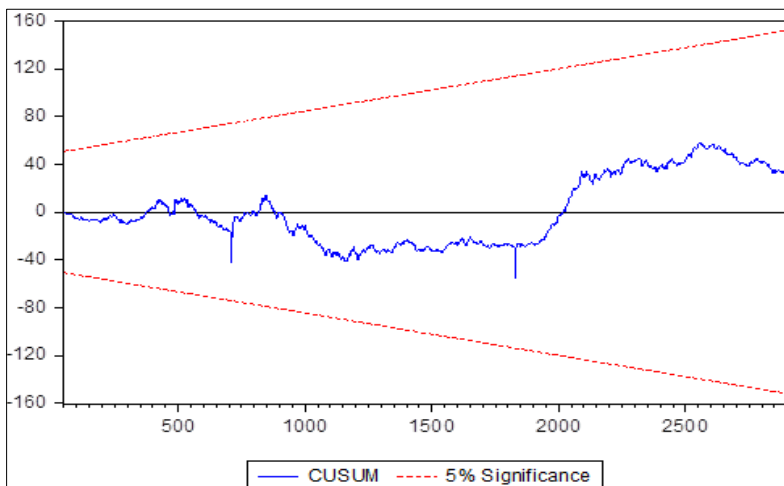
Error correction Model (ECM)

The ECM delivers a background for establishing links among the short-run and long-run methodologies to econometric modeling. The outcome of the gold prices ECM is presented in Table 4.

Table 4*Error Correction Mechanism outcomes*

Variable	Coefficient	Standard error	P-Value
C	8.136348	0.004776	0.0000*
GOLD (-1)	0.000422	0.000527	0.4230
GOLD (-2)	0.000225	0.000528	0.6698
GOLD (-3)	0.000121	0.000528	0.8190
GOLD (-4)	-2.71E-05	0.000529	0.9591
GOLD (-5)	0.000480	0.000530	0.3651
GOLD (-6)	0.001344	0.000531	0.0115*
GOLD (-7)	0.000415	0.000532	0.4361
GOLD (-8)	1.28E-05	0.000533	0.9808
GOLD (-9)	7.68E-05	0.000533	0.8854
GOLD (-10)	4.54E-05	0.000533	0.9321
GOLD (-11)	-0.000553	0.000532	0.2984
GOLD (-12)	-0.000461	0.000531	0.3852
GOLD (-13)	-0.000153	0.000530	0.7725
GOLD (-14)	0.000272	0.000529	0.6063
GOLD (-15)	0.000545	0.000527	0.3011
GOLD (-16)	0.000179	0.000526	0.7340
GOLD (-17)	-0.000728	0.000524	0.1649
GOLD (-18)	-2.62E-05	0.000521	0.9598
DOLLAR (-1)	-0.121923	0.005373	0.0000*
DOLLAR (-2)	-0.051886	0.005847	0.0000*
DOLLAR (-3)	-0.018070	0.005937	0.0024*
DOLLAR (-4)	-0.013902	0.005867	0.0179*
DOLLAR (-5)	-0.005362	0.005385	0.3195
DOLLAR (-1)	-0.001001	0.000541	0.0643
ECT	-0.998024	0.000523	0.0000*
R ²	0.999954	Adj-R ²	0.999954
DW	2.000647		

In the Table 4 result in * shows the significance of the coefficients at 5% level of significance. The results indicating significant impact on gold price and lagged changes and long run. The estimation outcome in table 4.4 shows that ECT has negative and statistically significant, so the coefficient of the ECT is -0.998024 suggests reasonable adjustment process. About 99 percent of the disequilibrium of the previous period is adjusted back to the long run equilibrium in the current period. The significance of lagged equilibrium correction term with negative sign confirms the convergence of long run equilibrium. Plot of cumulative sum of recursive residuals of the ARDL model to check for the stability of the estimated parameters. The value of Durbin Watson test is equal to 2.000647 which indicate no autocorrelation.

Figure 4*Plot of Cumulative Sum***Figure 5***Plot of Cumulative Sum of Squares*

The statistics of cumulative sum (CUSUM) and the cumulative sum of squares (CUSUMsq) are shown in Figures-4 and 5. straight lines represent critical bounds at 5 percent significance level. The results indicate that both graphs are found between the critical bounds at 5 per cent level of significance. The results displayed that the model is stable in the meantime the critical bounds at 5 percent fell in between 5 percent lines. The presence of co integration among the variables sets the stage for testing the Granger causality. This indicates that investment in gold is a hedge against inflation not only in the short run but also in the long run.

Table 5*ARMA (3, 1) Results*

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	7.867961	3.741934	2.102646	0.0356
AR (1)	1.000502	0.002583	387.3007	0.0000
AR (2)	-0.009740	0.004473	-2.177713	0.0295
AR (3)	0.009103	0.004345	2.095244	0.0362
MA (1)	-0.884357	0.002623	-337.1270	0.0000
SIGMASQ	0.008409	2.20E-05	383.0178	0.0000

R-squared	0.943708	Mean dependent var	8.149398
Adjusted R-squared	0.943611	S.D. dependent var	0.386577
S.E. of regression	0.091798	Akaike info criterion	-1.934511
Sum squared resid	24.38736	Schwarz criterion	-1.922154
Log likelihood	2811.041	Hannan-Quinn Criterion	-1.930059
F-statistic	9703.376	Durbin-Watson stat	2.000387
Prob(F-statistic)	0.000000		

Above table shows that at the model ARIMA (3, 1) P value give significant result, its mean that the model is good. R-square=0.943 indicate the model explain 94% variability of response data. The Akaike information criterion (AIC) is an estimator of the relative quality of statistical models for a given set of data.

Figure 6

Forecasting of Gold Price

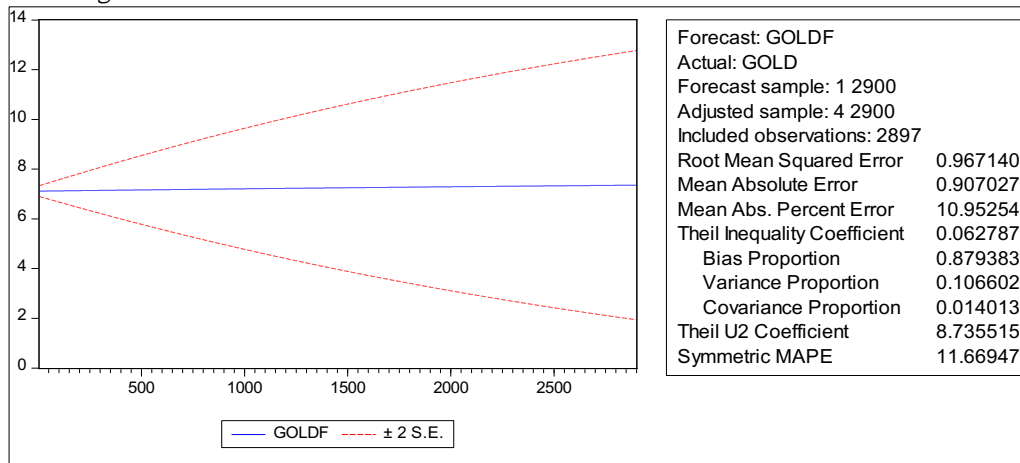


Figure 6 shows the forecast of ARIMA result indicate that AR (3) and MA (1) combinedly ARIMA (3,0,1,1) is a good model with 2S.E. It clearly indicates that Forecast of ARIMA (3,0,1,1) are better than other models.

Conclusion

The ultimate objective of this research was to check the pattern of stationary and non-stationary of daily gold price in Pakistan and estimate the long run and short run relationship between daily gold price of Pakistan and US-Dollar and also forecasting of daily gold price with relation US-Dollar. The data set used in this study consists of daily observations covering the period from 2007 to 2018. The Gold price was taken as the response variable, and the explanatory variable is US-Dollar. The results revealed by employing augmented ducky Fuller (ADF) technique, the data related to gold price is stationary at level. $P = 0.0001$ which show stationary at level but US-Dollar non-stationary at level it becomes stationary at 1st order of co-integration. Figure 1st also shows graphically the stationary of gold price at level. Figure 2 show graphically that US-Dollar is nonstationary at level. The pattern reveals that there are many ups and downs in the US-Dollar.

It becomes stationary at first difference which can be show at figure 3. To test the long-run and short-run relationship, we apply ADF and ARDL model above table 4-4 the value of Error correlation model (ETC) = -0.9998 is negative and also statistically significant, which indicate that long run relation exists between variables. The value of Durbin Watson test=2.0064 which also indicate no autocorrelation occur between variables. Last figure indicates stability of variable result show that the model is stable at 5% critical bound. Autoregressive integrated moving average (ARIMA) is one of the good models used for foresting purpose it also indicates the good model current study shows that ARIMA (3,0,1,1) is a good model. Which shows that AR (3), MA (1) gold price co integrated at level but US-Dollar is co integrated at first difference.

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